Registration No.: | | | | | | | | | | |

Total Number of Pages: 02

Course: M.Sc.I
Sub_Code: FMCC801

8th Semester Regular Examination: 2024-25
SUBJECT: Algebra-II
BRANCH(S): M.Sc.I (MC)
Time: 3 Hours
Max Marks: 70
Q.Code: S006
Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)
The figures in the right hand margin indicate marks.

Part-I

Q1 **Answer the following questions :** **(2 x 10)**

- **a)** Define alternating group. What is the order of the group A_5.
- **b)** Is $Z_3 \oplus Z_5$ isomorphic to Z_{15}. Justify your answer.
- **c)** List the up to isomorphism of an abelian group of order 108.
- **d)** Find the number of subgroups of order 7 in $Z_7 \oplus Z_7$.
- **e)** Find the factorization of the polynomial $\mathrm{f}(x) = x^5 + 2x^2 + 2x + 2 \in Z_3[x]$. Check whether this factorization is reducible or irreducible.
- **f)** Show that $Q(\sqrt{2},\ \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$
- **g)** Find the splitting field of $x^n - a$ over Q.
- **h)** State fundamental theorem of Galois theory.
- **i)** Determine the group of field automorphism of $Q(\sqrt[4]{2})$ of Q.
- **j)** Determine the irreducible factorization of $x^6 - 1$ over Z_2.

Part-II

Long Answer Type Questions (Answer Any five)

Q2 **a)** A subgroup N of a group G is a normal subgroup of G iff the product of two right cosets of N in G is again a right coset of N in G. **(5 + 5)**

b) State and prove first isomorphism theorem.

Q3 **a)** If ϕ is homomorphism of G into $\overline{G}$ with kernel K, then K is a normal subgroup of G. **(5 + 5)**

b) Show that every principal ideal domain is a unique factorization domain.

Q4 **a)** If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H. **(5 + 5)**

b) Find the minimal polynomial for $\sqrt[3]{2} + \sqrt[3]{4}$ over Q.

Q5 **a)** A polynomial $f(x)$ over a field F has a multiple zero in some extension E if and *only if* $f(x)$ and $f'(x)$ have a common factor of positive degree in $F[x]$ **(5 + 5)**

b) Let F be a field and let $f(x) \in F[x]$. Then show that two splitting fields of $f(x)$ over F are isomorphic.

Q6 **a)** Show that $a\ finite\ normal$ extension K of a field F is cyclic over F if G(K/F) is cyclic. **(5 + 5)**

b) If K is an algebraic extension of E and E is an algebraic extension of F, then show that K is an algebraic extension of F.

Q7 **a)** Determine the Ga$lois$ group of $x^3 - 1$ over Q and $x^3 - 2$ over Q. **(5 + 5)**

b) Let ω be a primitive nth root of unity. Then show that $Gal\left(\frac{Q(\omega)}{Q}\right) \approx U(n)$.

Q8 **a)** Let N be a normal subgroup of a group G. If both N and G/N are solvable, then show that G is solvable. **(5 + 5)**

b) For every positive integer n, show that $x^n - 1 = \prod_{d|n} \phi_d(x)$ where the product runs over all positive divisors d of n.